WE04 SIMPLE HARMONIC MOTION

SPH4U

EQUATIONS

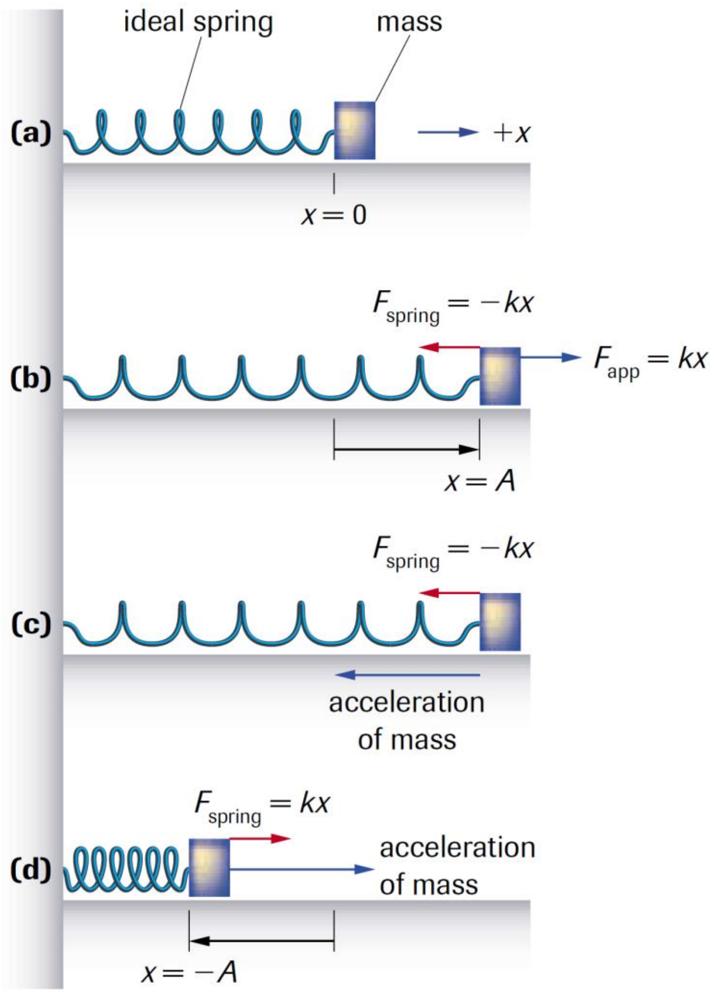
- Simple Harmonic Motion

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

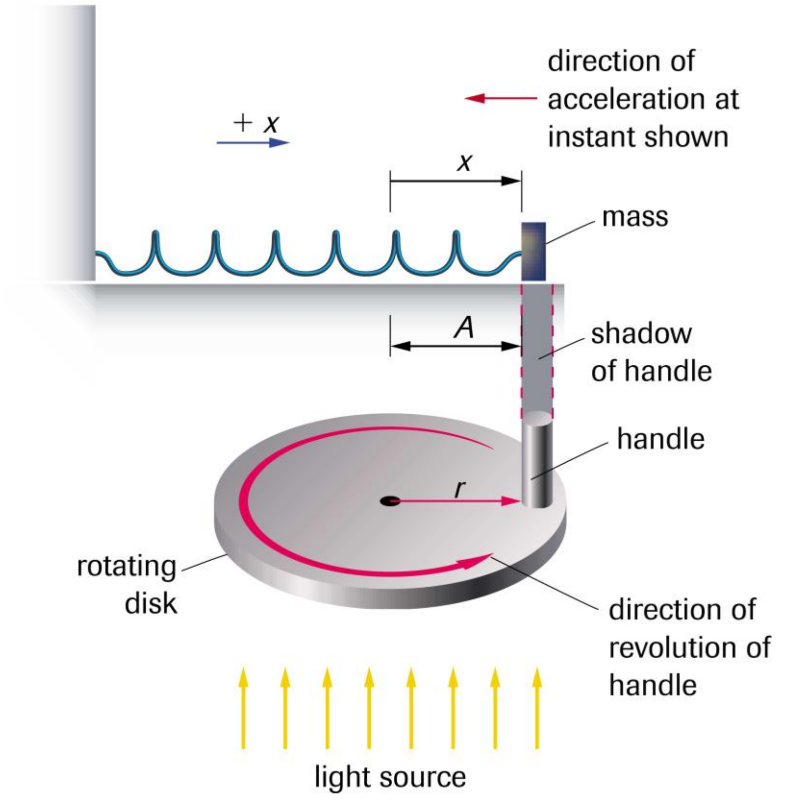
SIMPLE HARMONIC MOTION

- **Simple Harmonic Motion (SHM):** periodic vibratory motion in which the force (and the acceleration) is directly proportional to the displacement
- **Amplitude:** maximum displacement $(x = \pm A)$



SHM –CONT.

- We can model SHM with circular motion by combining Hooke's Law and Newton's 2nd law using a reference circle
- The motion of the spring system and the rotating disk are such that they have the same horizontal displacement from center
  - They are in sync, with the radius of the circle equal to the amplitude of SHM
- This allows us to use the equations for circular motion to model SHM



SHM – CONT.

- Recall, for circulation motion

$$a_c = \frac{4\pi^2 r}{T^2} \qquad \text{or} \qquad T = 2\pi\sqrt{\frac{r}{a_c}}$$

- Since $r = A$ for the reference circle, we get

$$T = 2\pi\sqrt{\frac{A}{a_c}}$$

- NOTE: the acceleration is not constant, so we need to find a more general equation

SHM – CONT.

- Hooke's Law: $F_x = -kx$
- Newton's 2nd Law: $F_x = ma_x$
- By equating the two laws, we get

$$ma_x = -kx$$

$$a_x = \frac{-kx}{m}$$

  - This shows that $a_x \propto x$, since k and m are constants

SHM – CONT.

- We can rearrange the equation further to get

$$\frac{-x}{a_x} = \frac{m}{k} = \text{constant}$$

- From our previous equation for T, $\frac{A}{a_c}$ is one value of $\frac{-x}{a_x}$, which gives us the more general equation

$$T = 2\pi\sqrt{\frac{-x}{a_x}} \quad \text{or} \quad \boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}}$$

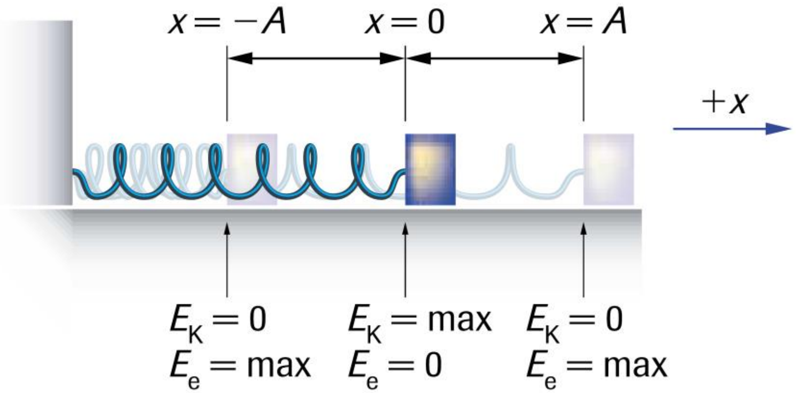
- Using replacing period with frequency, we get

$$\boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

ENERGY IN SHM

- In SHM, we have a maximum E_e when $x = \pm A$, and a minimum when $x = 0$
- The total energy at any time in SHM is given by

$$E_T = E_e + E_K$$
$$E_T = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$



PROBLEM 4

A 55-g box is attached to a horizontal spring of force constant 24 N/m. The spring is then compressed to a position $A = 8.6$ cm to the left of the equilibrium position. The box is released and undergoes SHM.

(a) What is the speed of the box when it is at position $x = 5.1$ cm from the equilibrium position?

(b) What is the maximum speed of the box?

PROBLEM 4 – SOLUTIONS

(a) We use the prime symbol (′) to represent the final condition. We apply the law of conservation of mechanical energy at the two positions of the box, the initial position A and the final position x'.

$A = 8.6 \text{ cm} = 0.086 \text{ m}$

$m = 55 \text{ g} = 0.055 \text{ kg}$

$x' = 5.1 \text{ cm} = 0.051 \text{ m}$

$k = 24 \text{ N/m}$

$v' = ?$

$$E_T = E_T'$$

$$E_e + E_K = E_e' + E_K'$$

$$\frac{kA^2}{2} + 0 = \frac{kx'^2}{2} + \frac{mv'^2}{2}$$

$$kA^2 = kx'^2 + mv'^2$$

$$v' = \sqrt{\frac{k}{m}(A^2 - x'^2)} \quad \text{(discarding the negative root)}$$

$$= \sqrt{\frac{24 \text{ N/m}}{0.055 \text{ kg}}\left((0.086 \text{ m})^2 - (0.051 \text{ m})^2\right)}$$

$$v' = 1.4 \text{ m/s}$$

The speed of the box is 1.4 m/s.

PROBLEM 4 – SOLUTIONS

(b) The maximum speed occurs when $x' = 0$.

$$v' = \sqrt{\frac{k}{m}(A^2 - x'^2)}$$

$$= \sqrt{\frac{24 \text{ N/m}}{0.055 \text{ kg}} (0.086 \text{ m})^2}$$

$$v' = 1.8 \text{ m/s}$$

The maximum speed of the box is 1.8 m/s.

OTHER SHM

- Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

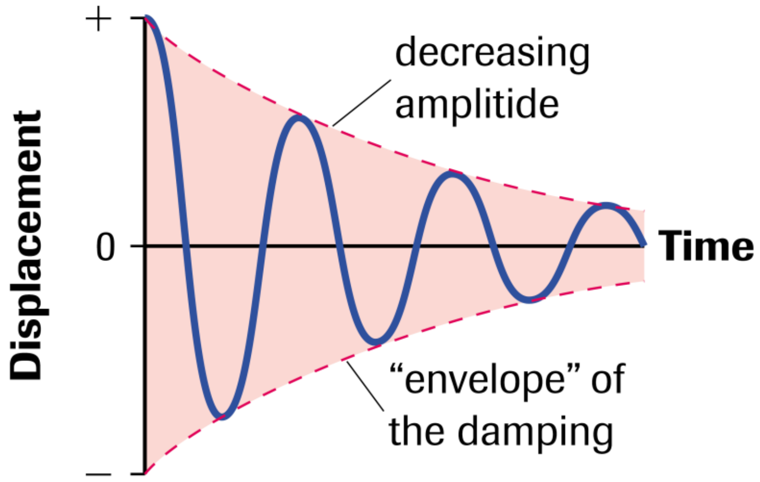
  - l – length of the string (radius of curvature) [m]
  - g – acceleration due to gravity [m/s^2]

Also,

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

DAMPED SHM

- In most mechanical systems, we do not want SHM
  - We want the system to settle down and come to rest
- **Damped Harmonic Motion:** periodic or repeated motion in which the amplitude of vibration and the energy decrease with time



SUMMARY

- Simple harmonic motion (SHM) is periodic vibratory motion such that the force (and thus the acceleration) is directly proportional to the displacement.
- A reference circle can be used to derive equations for the period and frequency of SHM.
- Damped harmonic motion is periodic motion in which the amplitude of vibration and the energy decrease with time.

PRACTICE

Readings

- Section 4.5, pg 203

Questions

- pg 218 #3,4,15